AP Stat- 4.5 notes $P(B|A)$

BAYES' RULE (aka Multistage Problems)

- Let A = 1^{st} event
- Let B = 2^{nd} event

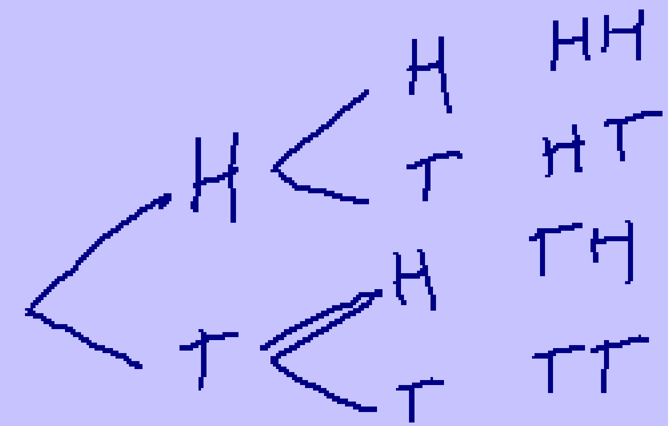
We know that P(B|A) ≠ P(A|B) = Prob. of 1^{st} event, know 2^{nd} event

So how can we find P(A|B)? In other words, how can we find the probability that the first event (A) happened given that we know the second event (B) happened?

Bayes' Rule: (page 356)

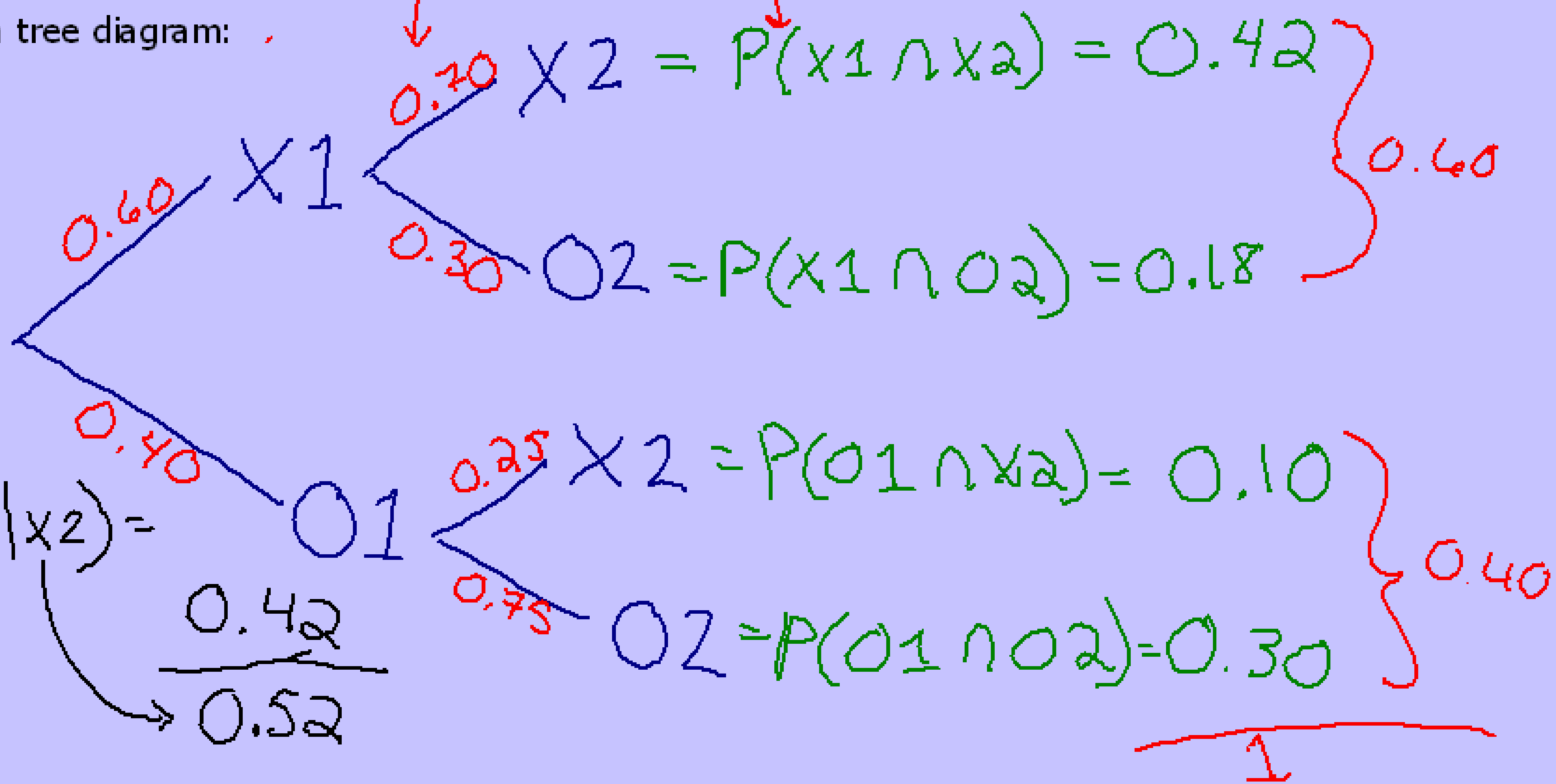
P(A|B) = ________________________

However, this rule is long, complicated, and a very difficult formula.
So instead, we use **TREE DIAGRAMS**



Think of this example:
A basketball player shoots 2 free throws. The following probabilities apply:
Prob. of making the 1st = 0.6 $P(X1)$
Prob. of making the 2nd **given that** you make the 1st = 0.70 $P(X2 \mid X1)$
Prob. of making the 2nd **given that** you missed the 1st = 0.25 $P(X2 \mid O1)$
conditional and's

Create a tree diagram:



$P(X1 \mid X2) = \dfrac{0.42}{0.52}$

So now lets answer some questions:

1. What is the probability of missing the 1st free throw?

2. What is probability of making the first free throw and making the second free throw?

3. What is the probability of making the second given that you missed the first?

4. What is the probability of missing the second given that you made the first?

Now lets try some harder ones: *(use the probability rules!!)*

1. What is the probability of making the second free throw?

2. What is the probability of missing the second free throw?

So lets say you went to the bathroom and didn't see the first free throw......

3. What is the probability you made the first given that you make the second?

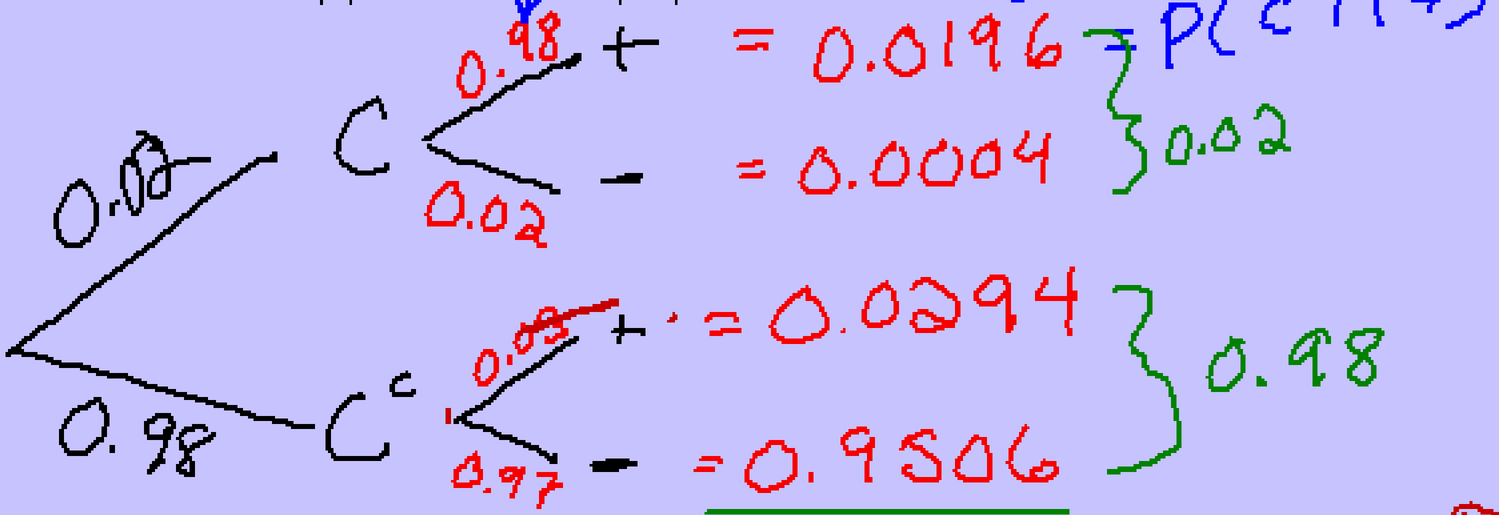
4. What's the probability you missed the first given that you make the second?

Example #1

CONDITIONAL

AND

- *A cancer clinic gives free cancer test (hypothetically)*
- It is known that 2% of the people that come into the clinic have cancer *(hypothetically)*
- It is known the test comes up positive in 98% of people with cancer
- It is known the test comes up positive in 3% of people without cancer



Answer the following questions:

1. What is the probability that someone tests positive given that they have cancer? $0.98 = P(+ | c)$
2. What is the probability that someone tests positive given that they don't have cancer? $0.03 = P(+ | c^c)$
3. What is the probability that someone tests negative given that they have cancer? $P(- | c) = 0.02$
4. What is the probability that someone tests negative given that they don't have cancer? $P(- | c^c) = 0.97$
5. What is the probability that someone tests positive? Negative? $P(+) = 0.049$ $P(-) = 0.951$
6. What is the probability that someone has cancer given that they test positive? *(This is called the accuracy of the test)*
7. What is the probability that someone doesn't have cancer given that they test positive? *(this is called a false positive)* $P(C | +) = \frac{0.0196}{0.049} = 0.40$
8. What is the probability that someone has cancer given that they test negative?

$P(c^c | +) = \frac{0.0294}{0.049} = 0.6$

$P(C | -) = \frac{0.0004}{0.951} = 0.000421$

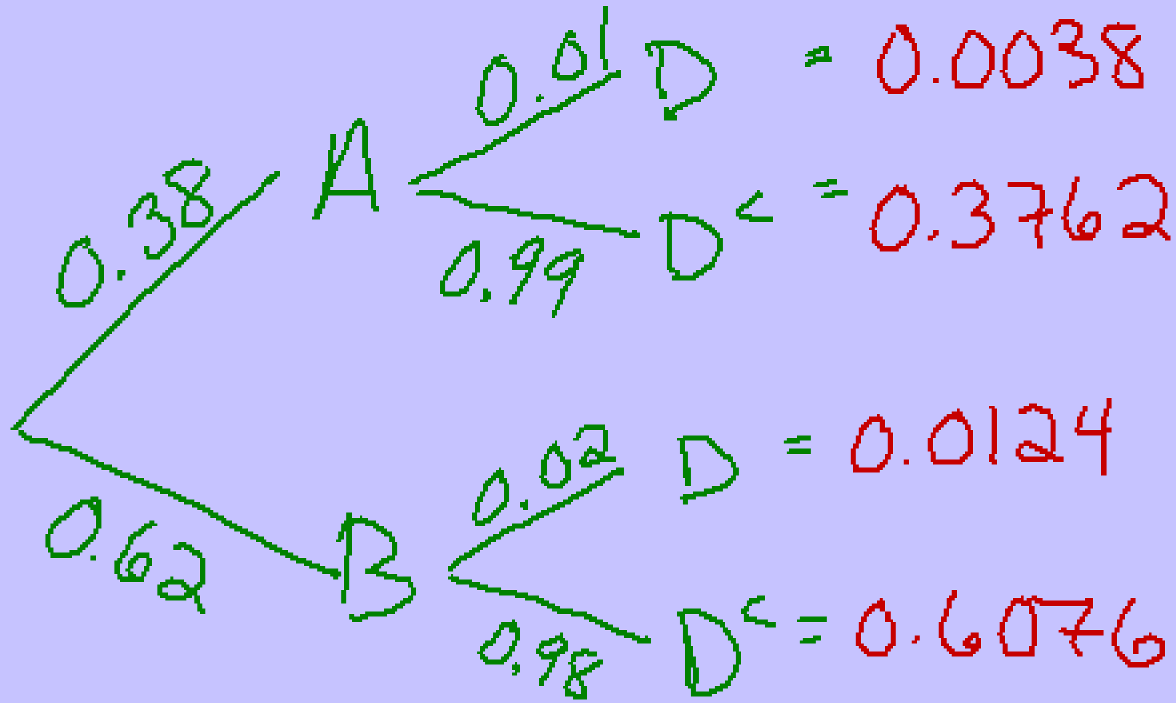
Example #2:

- There are 2 textbook making companies, A and B
- It is known that 1% of company A's books are defective
- It is known that 2% of company B's books are defective
- CB South gets 38% of its books from company A and the rest from company B

$P(D|A) = 0.01$

$P(D|B) = 0.02$

$P(A) = 0.38$



Questions:

1. *What is the probability that a book is NOT defective?*

$P(D^c) = 0.3762 + 0.6076 = 0.9838$

2. *If a book is not defective, what's the probability that it came from company B?*

$P(B|D^c) = \frac{0.6076}{0.9838} = 0.6176$

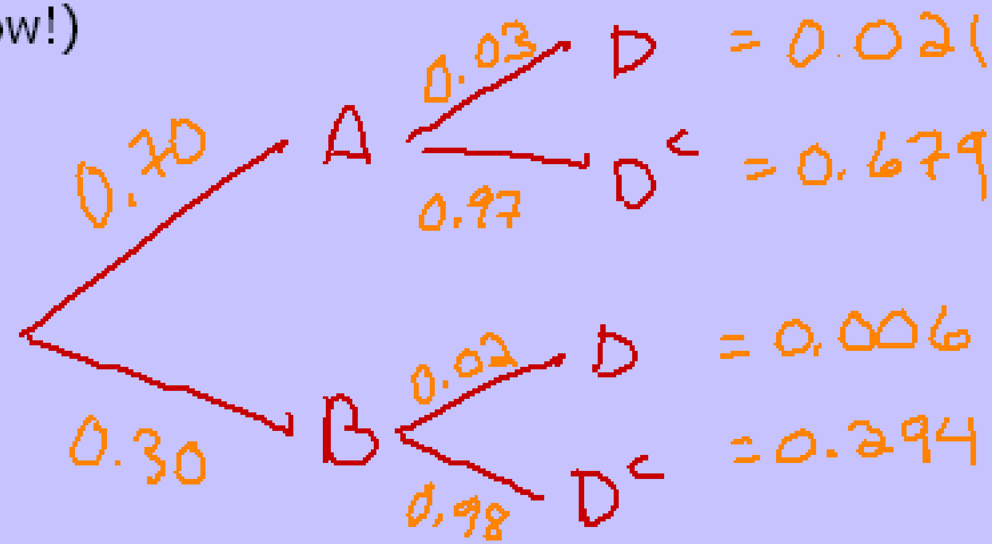
3. *If we open a book and it **IS** defective, what's the probability that is from company A? Company B?*

$P(A|D) = \frac{0.0038}{0.0162} = 0.2346$

$P(B|D) = \frac{0.0124}{0.0162} = 0.7654$

Try these two problems.

A VCR manufacturer receives 70% of its parts from factory A and the rest from factory B. Suppose that 3% of the output from A are defective, while only 2% of the output from B are defective. (make a diagram below!)



0.021 + 0,06

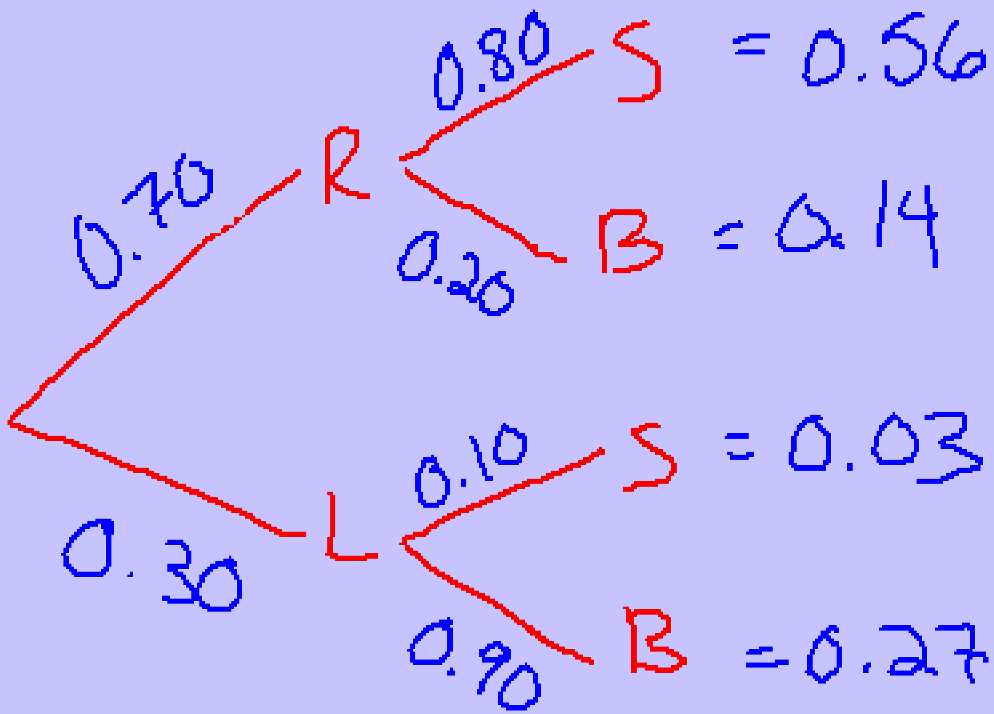
What is the probability that a received part is defective? $P(D) = 0.027$

If a randomly chosen part is defective, what is the probability that it came from factor A? From factory B?

$$P(A \mid D) = \frac{0.021}{0.027} = 0.778$$

$$P(B \mid D) = \frac{0.006}{0.027} = 0.222$$

A particular football team Is known to run 30% of its plays to the left and 70% to the right. A linebacker on an opposing team notes that the right guard shifts his stance most of the time (80%) when plays go to the right and that he uses a balanced stance the rest of the time. When plays go left, the guard takes a balanced stance 90% of the time and the shift stance the remaining 10%. What is the probability that a play will go to the left if the guard is balances? (make a diagram below!)

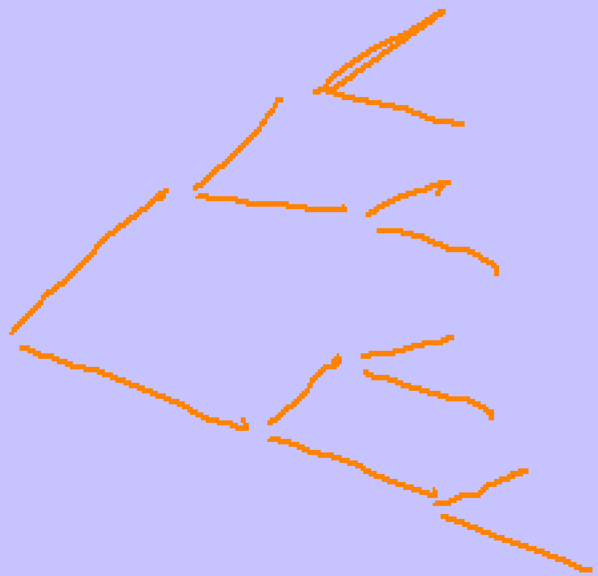


$$P(L|B) = \frac{0.27}{0.14 + 0.27}$$

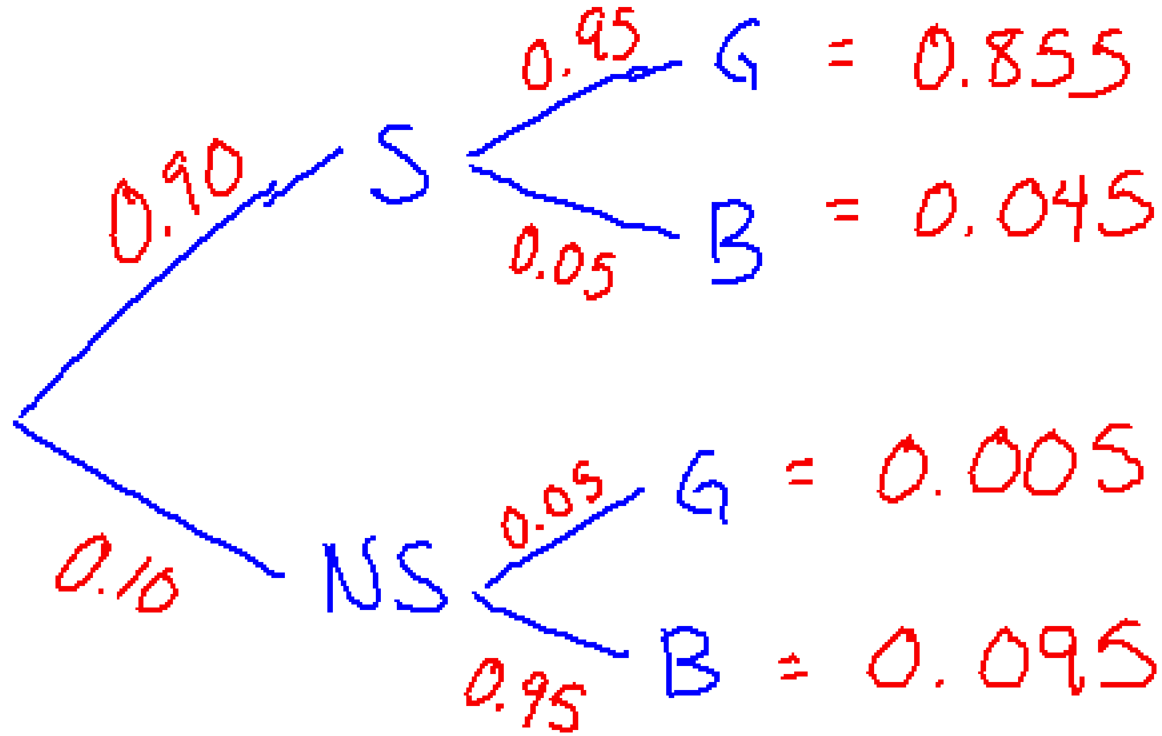
$$= 0.659$$

Try the worksheet with the Multistage problems on it.

#1, 3 (2 if you are ambitious)



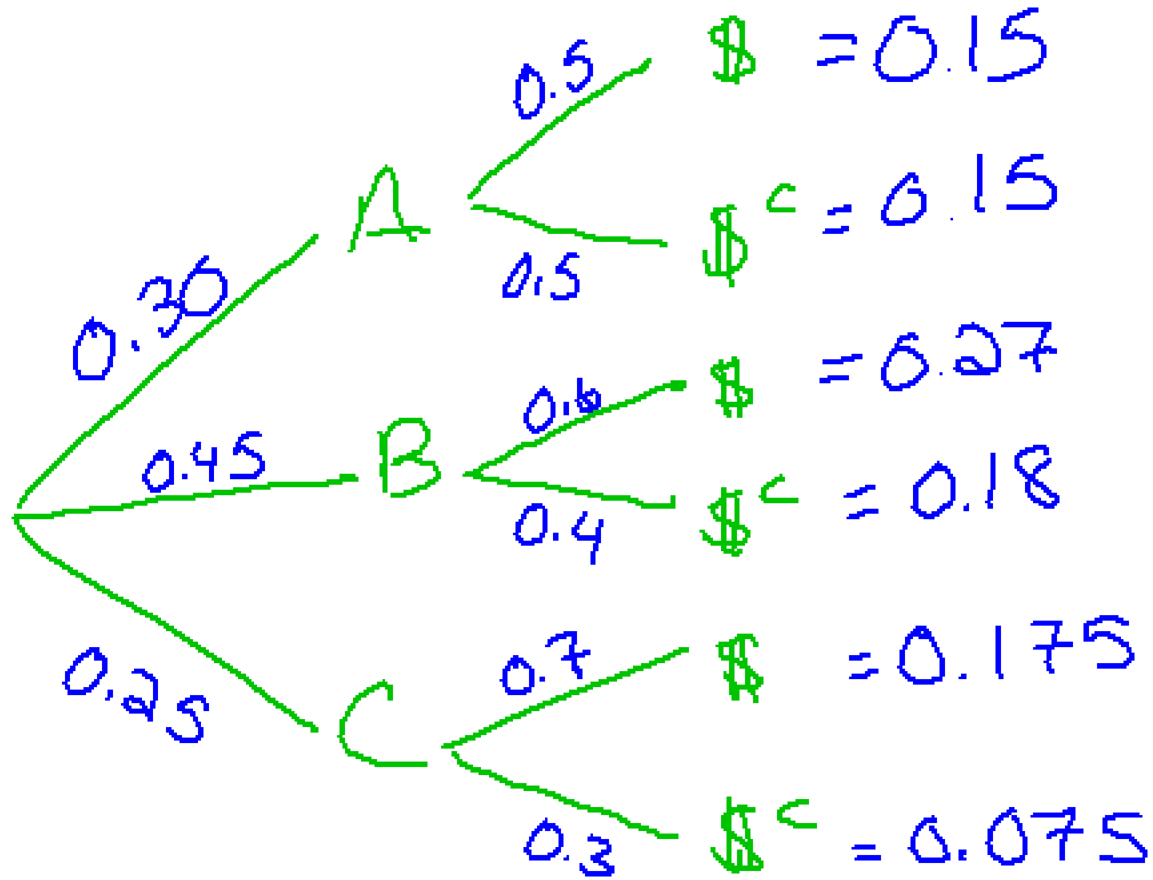
1



a) $P(G) = 0.86$

b) $P(S|G) = \dfrac{0.855}{0.86} = 0.994$

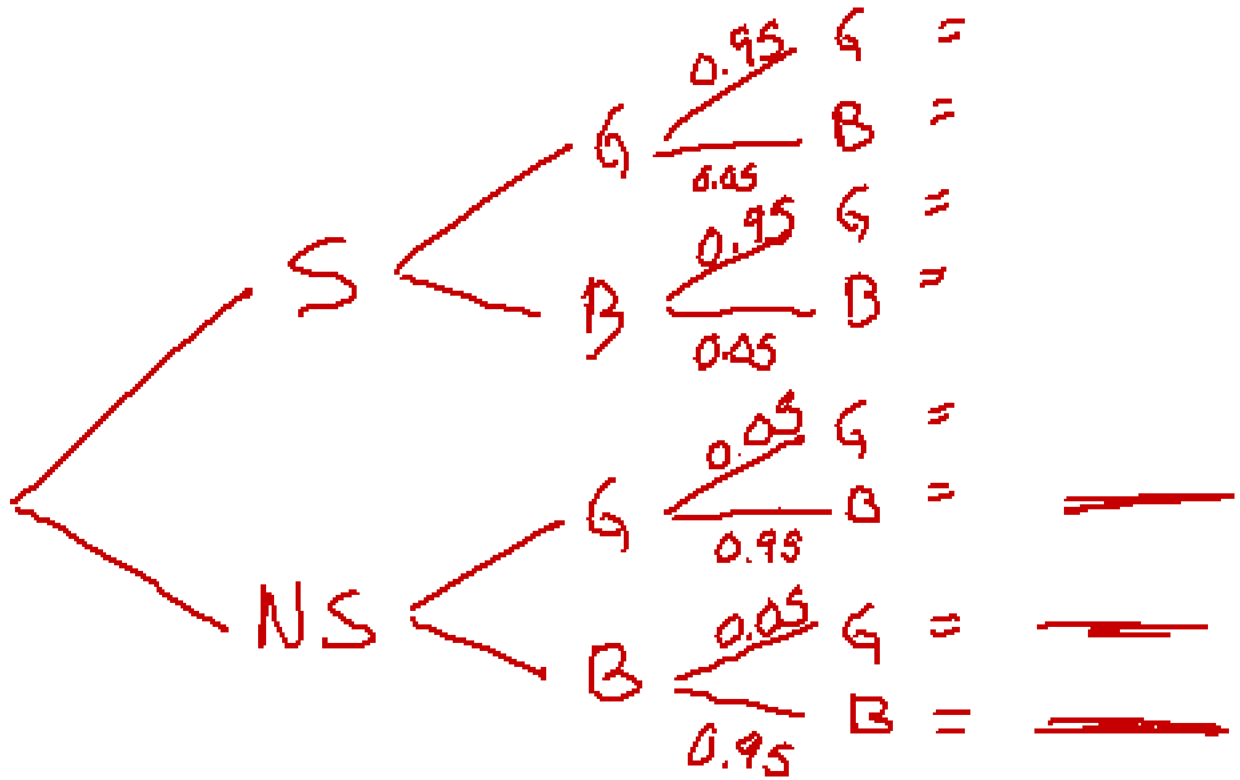
3



a) $P(\$) = 0.15 + 0.27 + 0.175 = 0.595$

b) $P(A \mid \$) = \dfrac{0.15}{0.595} = 0.252$

2



a) $P(NS \cap G \cap G) = 0.00025$

b) $P\left(\begin{matrix} G \cap B \\ B \cap G \\ B \cap B \end{matrix} \middle| NS\right) = \dfrac{}{0.1} = 0.9975$